IRKGL16: Implicit RK Solver For High Precision Numerical Integration

Context

- We focus on solving **non-stiff ODEs** with **high accuracy**
- Most numerical integration of ODEs is performed with double-precision arithmetic (64-bit) but some require **extra precision**
- Examples: chaotic problems, integration over long time periods,...
- julia language supports **arbitrary precision number systems** for fast solving with high accuracy

Motivation

- Develop a new solver written in Julia
  - **Contribution** to DifferentialEquations.jl ecosystem
- Show that Implicit methods can be **more efficient** than explicit ones
- IRKGL16 implicit solver (an 8-stage IRK scheme based on Gauss-Legendre nodes) can take advantage of **modern computer technology**:
  - Parallelism
  - Mixed-precision arithmetic

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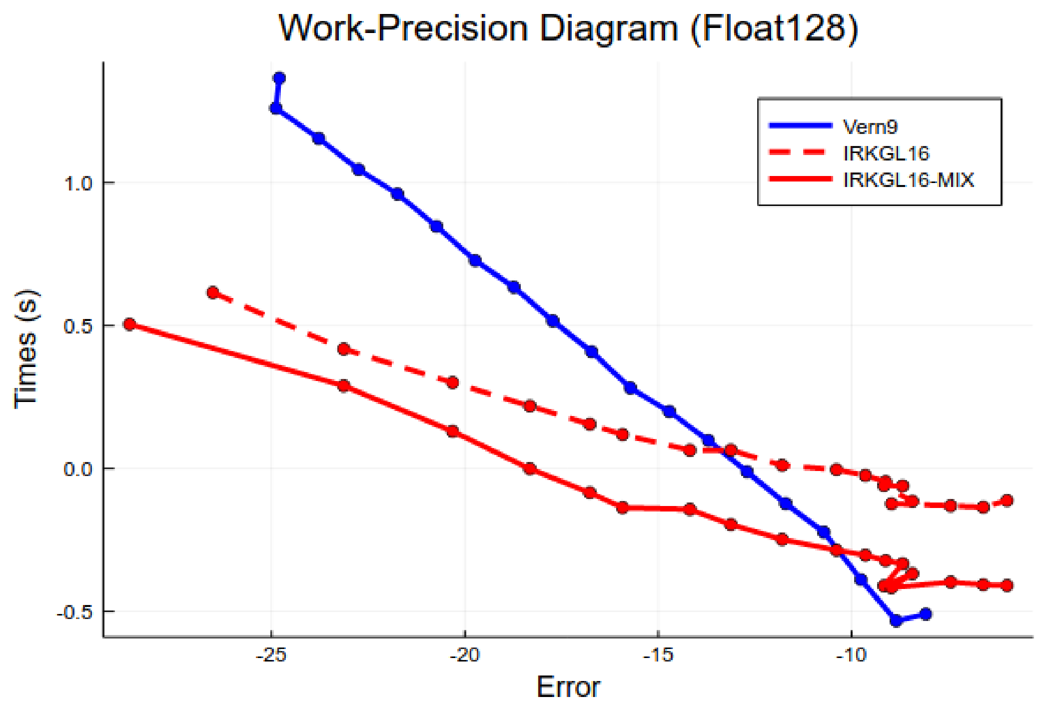
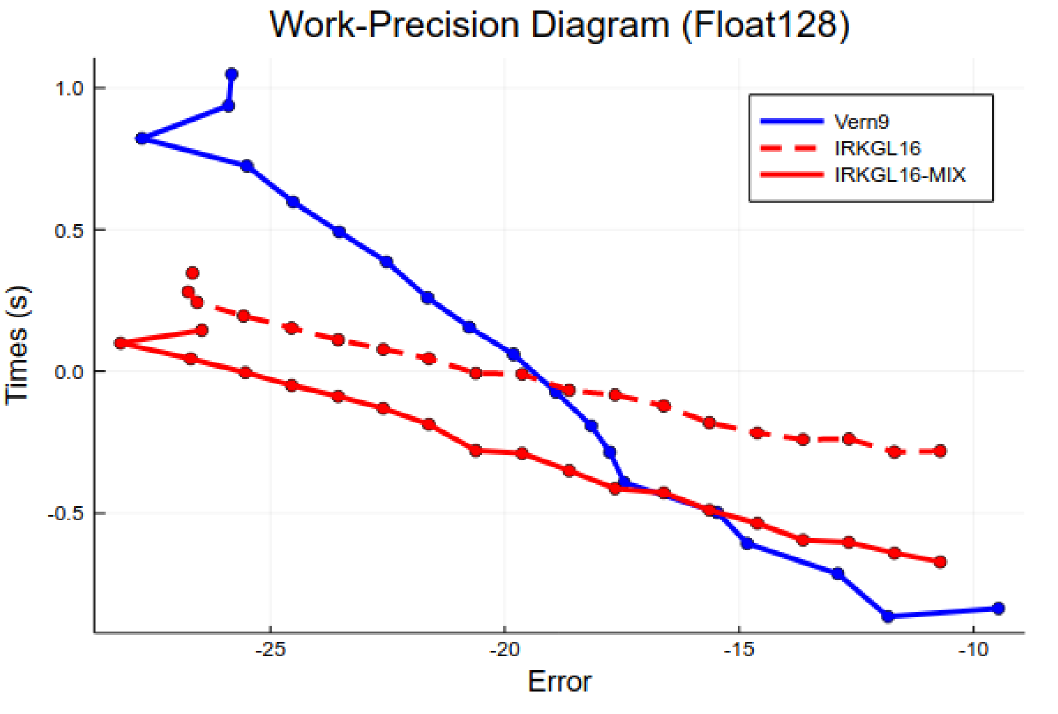
General First Order ODE Problems $\frac{du}{dt} = f(u,t)$

Challenge: improve efficiency of DifferentialEquations.jl's **Vern9** algorithm

Discrete NonLinear Schrödinger (NLS) with N=5



Restricted Three Body Problem (RTBP)



Conclusions from experiments

- **IRKGL16** is more efficient than **Vern9** for high precision computations
- Mixed-precision arithmetic improves performance of IRKGL16 solver

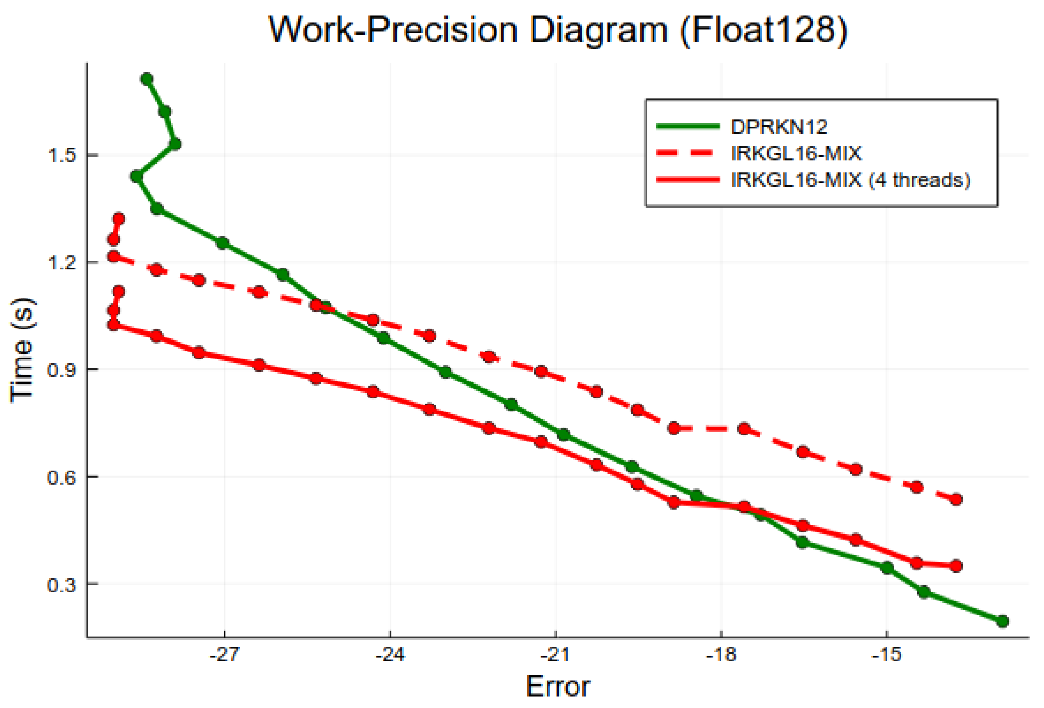
  Combining quadruple precision (128-bit) arithmetic with double precision arithmetic (64-bit)

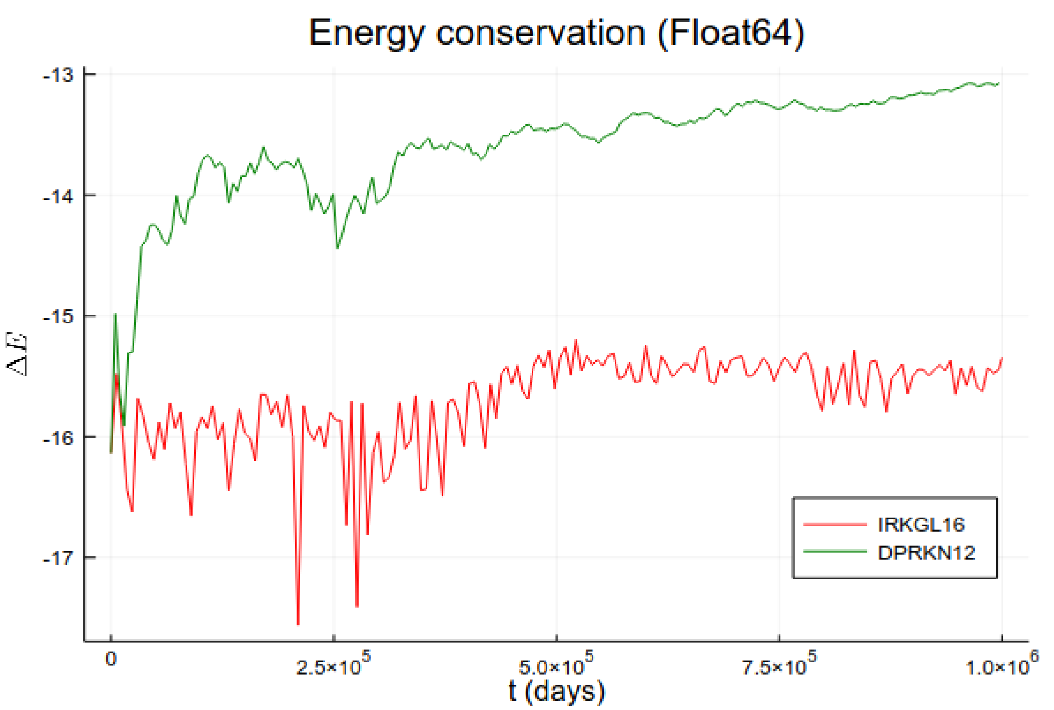
Second Order ODE Problems $u = (q, v), \quad \frac{dq}{dt} = f(v), \quad \frac{dv}{dt} = g(q,t)$

Challenge: improve efficiency of DifferentialEquations.jl's **DPRKN12** algorithm

Pleiades Problem (PLEI)

Solar-System Problem (SSP)





Conclusions from experiments

- PLEI: **IRKGL16-MIX (4 threads)** is more efficient than **DPRKN12** for high precision computations

  Multithreading version of IRKGL16 solver based on stage-wise parallelization
- SSP: **IRKGL16 fixed-step size** computation conserves energy better than **DPRKN12** (for comparable execution time) for long time integration

Summary

https://github.com/mikelehu/IRKGaussLegendre.jl

Conclusions

About julia

- Julia's high-level syntax allows good **productivity** and **well organized code**
- DifferentialEquations.jl ecosystem offers functionality for easy integration of **new methods**

About IRKGL16

- It is **competitive for high-accuracy computations** that exceeds double precision arithmetic (tol<1e-16)
- **Mixed-precision** arithmetic and **multihreading** enhance efficiency of the solver

Future work

- **Fully Integrate** as an algorithm on the common interface of DifferentialEquations.jl
- Implementation of parallelized implicit ODE solvers for **large ODE systems**

Acknowledgments

- This work has received funding from Department of Education of the Basque Government through **MATHMODE Research Group** (IT1294-19)
- Thanks to **Chris Rackauckas** for his help throughout this project