method

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Generalized potential with respect to star 1 is given by Equation 3.16 in PHOEBE Scientific Reference

$$\Omega = \frac{1}{\varrho} + q\left(\frac{1}{\sqrt{\delta^2 + \varrho^2 - 2\varrho\lambda\delta}} - \frac{\varrho\lambda}{\delta^2}\right) + \frac{1}{2}\mathsf{F}^2(1+q)\varrho^2(1-\nu^2) \tag{1}$$

To get star 2 (Equation 3.15 in P.S.R.)

$$\Omega' = \frac{\Omega}{q} + \frac{1}{2}\frac{q-1}{q} \tag{2}$$

where

- $\varrho = r/a$ where r is the radial distance originating from star 1 and a is the semi-major axis
- q is the mass ratio
- $\delta = D/a$ is the normalized instantaneous separation between the two stars
- F is the synchronicity parameter (will be introduced when needed)
- λ and ν are the x and z directional cosines, respectively (Eqn. 3.8 P.S.R.)

The potential at the pole reduces to Equation 3.20 (PHOEBE Sci. Ref.)

$$\Omega = \frac{1}{\varrho_{\text{pole}}} + q\left(\frac{1}{\sqrt{\delta^2 + \varrho_{\text{pole}}^2}}\right) \tag{3}$$

The position of L_1 is given by Equation 3.23 (PHOEBE Sci. Ref.)

$$x_{L1} = z - \frac{1}{3}z^2 - \frac{1}{9}z^3 + \frac{58}{81}z^4 \tag{4}$$

where $z = (\mu/3)^{1/3}$ and $\mu = M_2/(M_1 + M_2)$

Before plugging back in to the equation of the general potential lets note the following:

- along the x-axis so $\nu = 0$ and $\lambda = 1$
- $\varrho = x_{L1}$
- lets check at periastron so $\delta = (1 - \varepsilon)$

$$\Omega = \frac{1}{x_{L1}} + q\left(\frac{1}{\sqrt{\delta^2 + x_{L1}^2 - 2x_{L1}\delta}} - \frac{x_{L1}}{\delta^2}\right) + \frac{1}{2}\mathsf{F}^2(1+q)x_{L1}^2 \tag{5}$$

We adopt the condition that

$$\mathsf{F} = \begin{cases} 1 & \text{if } \varepsilon < 0.05 \\ \sqrt{\frac{1+\varepsilon}{(1-\varepsilon)^3}} & \text{if } \varepsilon \geq 0.05 \end{cases} \tag{6}$$